

3.10 Sum & Difference Identities for Cosine

Name _____ Date _____ Period _____

Find the exact values (leave in terms of π) of the following sums or differences.

1. $\frac{3\pi}{4} + \frac{\pi}{3}$

2. $\frac{\pi}{4} - \frac{\pi}{3}$

3. $\frac{\pi}{2} - \frac{\pi}{6}$

Express each given angle as $\alpha + \beta$ or $\alpha - \beta$, where $\cos \alpha$ and $\cos \beta$ are known exactly.

4. 75°

5. 165°

6. $\frac{\pi}{12}$

7. $\frac{5\pi}{12}$

Use appropriate identities to find the exact value of each expression.

8. $\cos(15^\circ)$

9. $\cos(105^\circ)$

10. $\cos(165^\circ)$

11. $\cos\left(\frac{13\pi}{12}\right)$

12. $\cos\left(\frac{-\pi}{12}\right)$

13. $\cos\left(\frac{-5\pi}{12}\right)$

Simplify each expression by using the appropriate identities. Do not use a calculator.

14. $\cos(23^\circ)\cos(67^\circ) + \sin(23^\circ)\sin(67^\circ)$

15. $\cos(34^\circ)\cos(13^\circ) + \sin(34^\circ)\sin(13^\circ)$

16. $\cos(5)\cos(6) - \sin(5)\sin(6)$

$$17. \cos\left(-\frac{\pi}{2}\right)\cos\left(\frac{\pi}{5}\right) + \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{5}\right)$$

$$18. \cos(3y)\cos(y) - \sin(3y)\sin(y)$$

Match each expression with an equivalent expression from a-d.

$$19. \sin(20^\circ)$$

$$20. \cos(90^\circ)$$

$$21. \sec\left(\frac{\pi}{6}\right)$$

$$22. \sin\left(\frac{5\pi}{12}\right)$$

$$a) \csc\left(\frac{\pi}{3}\right)$$

$$b) \cos(70^\circ)$$

$$c) \cos\left(\frac{\pi}{12}\right)$$

$$d) \sin(0^\circ)$$

Simplify each expression by applying the odd/even identities, cofunction identities, and cosine of a sum or difference identities. Do not use a calculator.

$$23. \cos(10^\circ)\cos(20^\circ) + \sin(-10^\circ)\cos(70^\circ)$$

$$24. \sin(85^\circ)\sin(40^\circ) + \sin(-5^\circ)\sin(-50^\circ)$$

Solve each problem

$$25. \text{ Find the exact value of } \cos(\alpha + \beta) \text{ if } \sin \alpha = \frac{3}{5} \text{ and } \sin \beta = \frac{5}{13}, \text{ with } \alpha \text{ in quadrant II and } \beta \text{ in quadrant I.}$$

$$26. \text{ Find the exact value of } \cos(\alpha - \beta) \text{ if } \sin \alpha = \frac{\sqrt{3}}{2} \text{ and } \cos \beta = \frac{-\sqrt{2}}{2}, \text{ with } \alpha \text{ in quadrant I and } \beta \text{ in quadrant II.}$$

Write each expression as a function with α alone.

27. $\cos\left(\frac{\pi}{2} + \alpha\right)$

28. $\cos(180^\circ - \alpha)$

Verify that each equation is an identity.

29. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \tan x$

30. $\frac{\cos(x + y)}{\cos x \cos y} = 1 - \tan x \tan y$

31. $\cos(2x) = \cos^2 x - \sin^2 x$ Hint: $2x = x + x$

Simplify.

32. $\frac{\csc x}{\sec x}$

33. $(1 - \sin x)(1 + \sin x)$